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LiMoNAED: A LIMITED MOTION, NON-SHADING, ASYMMETRIC, ECLIPTIC-TRACKING DISH

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Abstract—The conventional design of a parabolic dish for a small solar conversion system places the receiver along the line between the center of the dish and the sun. This forces the receiver to move in a large arc during tracking, and produces some shading of the dish. In some applications, such large movement of the receiver is not acceptable. A new concentrator design is proposed for small systems with a constraint of limited mobility of the receiver. This is accomplished by using a first polar axis and a second axis that is aligned with the normal to the ecliptic plane. The new design features limited motion of the receiver, with inclination changing only within ±23.45°; off-axis reflector to eliminate shading; constant rotation speed in both axes; and constant flux distribution on the receiver.

1. INTRODUCTION

An effective way of collecting and concentrating sunlight is pointing a reflector at the sun, tracking the apparent motion of the sun such that the reflector remains aimed at the sun throughout the day, and placing a receiver at the focus of the reflector to convert the concentrated radiation into a useful form of energy. This is implemented in the common design of an on-axis parabolic dish with a receiver suspended at the focus along the centerline. In this design, the receiver moves over a large arc during each day. Some applications, such as dish-Stirling systems (a dish equipped with a Stirling engine for electricity generation), are largely unaffected by this motion. However, if other applications are considered, several issues may arise that require different concentrator solutions.

The large motion of the receiver precludes fixed connections to other equipment on the ground. Any connection is required to pass through or near the center of rotation of the dish, and to be flexible enough to accommodate rotation of 180°. In a dish-Stirling system, the only connection is electrical, which easily fits these requirements. This may not be true for other applications. For example, if the receiver is to be connected to ground-based equipment by heavily insulated pipes that are not flexible, or the length of the pipes must be minimized, then the translation of the receiver in space should be minimized.

The change in receiver orientation relative to the direction of gravity is also large during tracking, from horizontal to vertical and back within each day. If the operation of the receiver is affected by the direction of gravity, for example through changes in natural convection, then it is desired to minimize these orientation changes. Some solar chemical receivers could fall into this category. Another problem arises if engines such as commercial gas turbines are installed in a dish-engine system (Bammert *et al*., 1981; Fujita *et al*., 1982; Buck *et al*., 1996). These engines contain bearings that are designed for horizontal operation, and may fail due to unexpected side-forces in tilted operation. Other aspects, for example the drain of lubrication fluid, may also be affected by sharp changes of orientation.

Another motivation for reduction in the receiver motion can be the support. A receiver can be quite heavy, and the mechanical support that is needed to hold it in place with sufficient rigidity under variable load can be massive and expensive. If the receiver is placed near the ground, and its movement restricted or eliminated, then this massive structural element can be simplified.

Another issue in on-axis dish design is shading of the reflector by the receiver. This is not a

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problem in dish-Stirling systems where the receiver and engine are relatively compact. However, other applications may require larger elements at or near the focal region, and these could create a significant obstruction of radiation and reduction in system efficiency.

A possible solution is a stationary receiver with a tracking heliostat. However, the incidence angle on the heliostat changes considerably with time. The resulting optical aberrations and the variable cross-section that the heliostat presents to the sun (cosine effect) will cause significant time-dependent variations in the collected radiation. An astigmatic corrected heliostat (Zaibel *et al*., 1995) can be used, but it provides only a partial correction to this variation, particularly for short focal length.

The common design of solar furnaces also uses a tracking heliostat separate from the stationary receiver. In this case, the heliostat is usually flat and serves only to redirect the radiation towards another large reflector. The second reflector is stationary and usually parabolic, concentrating the radiation towards the receiver. The disadvantages of this arrangement are the doubling of the reflective area, and the variation of collected power during the day due to incidence angle on the heliostat.

The translation motion of the receiver can be eliminated in a standard dish by using a Cassegrainian geometry, with a hyperboloid reflector installed in front of the focal point, redirecting the radiation to a secondary focus that can be located at the dish center of rotation (Yehezkel *et al*., 1993; Feuermann *et al*., 1999). However, this arrangement does not reduce the required rotation. The receiver still needs to rotate over the full range of angles as in a standard dish design.

Another solution with a stationary receiver is the so-called ‘fix-focus’ dish (Mast and Mitzel, 1996). Daily tracking is performed around an axis that is parallel to the earth’s rotation axis, and a second motion corrects for seasonal variations in the sun’s declination. In this design, the target remains in a fixed position and the dish swings around it. This design requires the most massive element in the system to perform a significant translation motion rather than simple rotation around its center of mass, leading to considerable mechanical difficulty. The incidence angle on the dish changes with the seasonal variation, leading to changes in the flux distribution incident on the receiver. It is also necessary for the receiver to be axisymmetric about the polar axis in both the spatial and directional senses in order to accept the collected radiation, which is incident within an angular range that shifts during tracking.

We propose an alternative solution for a dish with a receiver that prefers to limit changes in position and orientation. The new design features the following.

- Limited motion of the receiver, with inclination changing only within ±23.45°.
- Off-axis design to eliminate shading of the concentrator by the receiver (this requires, however, increased reflector area).
- Time-invariant incidence angle, leading to a constant flux distribution on the receiver.
- Tracking of the dish at constant speed in both rotation axes.

The new design is labeled LiMoNAED: limited-motion, non-shading, asymmetric, ecliptic-tracking dish.

2. TRACKING THE SUN

Solar concentrating systems need to track the sun, i.e. constantly change their orientation in response to the changing direction toward the sun. Tracking usually relies on high-accuracy astronomical models. However, in the interest of clarity, a simplified model of the earth–sun geometry is used here. The same methodology can be applied to the more sophisticated models.

2.1. Direction towards the sun

The earth moves relative to the sidereal coordinate system fixed with respect to the stars, as shown in Fig. 1. The motion of the earth can be decomposed into an orbital motion O and a spin S. The orbital motion is approximately (within ±1.6% variation) a rotation around the ecliptic axis $\vec{e}$, which is fixed with respect to the sidereal coordinate system, with approximately constant angular velocity. If T_Y is the time needed for a complete orbital rotation, then the angular velocity is:

$$\omega_Y = 2\pi / T_Y. \tag{1}$$

We denote by $R(\vec{a}, \vartheta)$ a rotation around the axis $\vec{a}$ by the angle ϑ, and a vector $\vec{d}$ is transformed by rotation according to: $\vec{d}\,' = R(\vec{a}, \vartheta) \cdot \vec{d}$. The orbital motion around the ecliptic axis is then:

$$O = R(\vec{e}, \omega_Y t). \tag{2}$$

The sidereal day T_D is the time in which stars apparently revolve in a complete circle around the

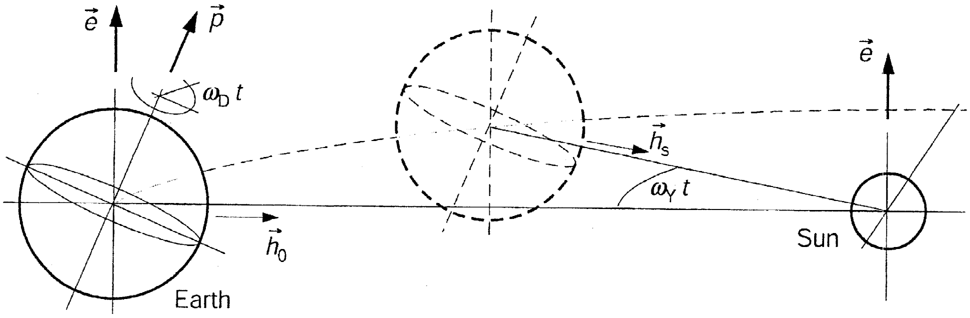

Fig. 1. Geometry of the earth orbital and spin rotations.

earth. It is related to the solar day T_{SD} by $T_D = T_{SD}(1 - 1/365.24)$. The spin S is a rotation around the polar axis $\vec{p}$ with constant angular velocity $\omega_D = 2\pi/T_D$:

$$S = R(\vec{p}, \omega_D t). \tag{3}$$

The polar axis $\vec{p}$ points from the South Pole to the North Pole, and is fixed in the sidereal coordinate system, tilted relative to the ecliptic axis by an angle of $\delta = 23.45°$.

The radius of the earth (6000 km) is negligible compared to the distance from the earth to the sun (150,000,000 km). Therefore, the direction from a location on earth towards the center of the sun is essentially independent of the spin movement. At time $t = 0$ this direction is denoted $\vec{h}_0$. The direction from the earth to the sun in the sidereal coordinate system, $\vec{h}_s$, varies according to:

$$\vec{h}_s(t) = O \cdot \vec{h}_0 = R(\vec{e}, \omega_Y t) \cdot \vec{h}_0. \tag{4}$$

An observer fixed with respect to the earth, i.e. in the Ptolemaic coordinate system, is subject to the spin rotation in addition to the orbital motion. The direction towards the sun as it appears to this observer, $\vec{h}_t$, is:

$$\vec{h}_t(t) = S(\vec{p}, \omega_D t) \cdot O(\vec{e}, \omega_Y t) \cdot \vec{h}_0. \tag{5}$$

2.2. Reorientation vs. 'pointing'

One can describe the full orientation of a three dimensional body by three independent difference vectors $\vec{d}_1$, $\vec{d}_2$, $\vec{d}_3$. Thus, a reorientation of a rigid body is a linear transformation M, which transforms these three vectors: $\vec{d}_i^{\,*} = M \cdot \vec{d}_i$. The transformation is subject to six constraints of the form $\vec{d}_i^{\,*} \cdot \vec{d}_j^{\,*} = \vec{d}_i \cdot \vec{d}_j$, expressing the fact that the body is rigid, i.e. distances and angles are not affected by the rotation. Orientation of a rigid body has therefore $3 \times 3 - 6 = 3$ degrees of freedom. Consequently, any reorientation of a three-dimensional rigid body can be realized by three consecutive rotations. More precisely, given three axes, one can find three angles such that a given orientation is achieved by the superposition of the three rotations:

$$M = R(\vec{a}_3, \vartheta_3) \cdot R(\vec{a}_2, \vartheta_2) \cdot R(\vec{a}_1, \vartheta_1). \tag{6}$$

Physically, a mechanism to achieve this must be composed of four parts, each part being a rigid body, the parts being interconnected by physical axes and bearings. The first part of the mechanism is fixed with respect to the reference coordinate system, e.g. the earth; the second is connected via a joint to the first, such that only the direction $\vec{a}_1$ is invariant, etc.

Most solar receivers that are candidates for use in a dish concentrator possess a rotational symmetry around an axis perpendicular to the receiver aperture. In this case, a general reorientation in space is not needed, and it suffices to align the axis of symmetry with a given direction. We refer to a change in orientation such that one difference vector associated with the rigid body coincides with a given direction as 'pointing' the system in that direction. For example, a telescope is pointed towards a star. Similarly, it is sufficient to point a dish solar concentrator towards the sun.

Pointing a device in a certain direction is represented by a linear transformation, which transforms only one vector:

$$\vec{d}_1^{\,*} = M \cdot \vec{d}_1 \tag{7}$$

subject to the constraint

$$\vec{d}_1^{\,*} \cdot \vec{d}_1^{\,*} = \vec{d}_1 \cdot \vec{d}_1. \tag{8}$$

Thus, $3 - 1 = 2$ degrees of freedom are needed in order to point a device into a given direction. This can be physically realized by two consecutive rotations around given axes. Highly concentrating solar systems generally incorporate two-axis tracking.

The motivation for the above discussion is to stress that two axes are not sufficient in general for a full reorientation of a device. Rather it only allows pointing it into a desired direction. Full reorientation is important for correction of astigmatism, since a correction can modify the curvature of the reflector selectively in the saggital and tangential planes (Zaibel *et al.*, 1995). The correction therefore breaks any symmetry of the reflector with respect to the 'pointing' axis and requires that the modified curvatures remain within the corresponding planes. This requires full reorientation of the reflector, including a rotation of the reflector in the plane perpendicular to the 'pointing' axis.

Pointing is sufficient for systems that track the sun as a single unit, for example a conventional solar dish concentrator (on- or off-axis) with a full-motion receiver that is mounted rigidly to the dish structure. In this case, the entire sun–reflector–receiver geometry is invariant during tracking. The 'fix-focus' dish design also provides full reorientation, since the polar tracking axis always remains within the tangential plane. In a completely split system such as a conventional heliostat with azimuth-elevation mount, the heliostat only points to the required direction but does not reorient to the correct planes. It is therefore not possible to correct the conventional heliostat for astigmatism. The choice of a target-aligned mount (Zaibel *et al.*, 1995) permits synchronization of the heliostat rotation with the rotation of the tangential plane, recovering the property of correct reorientation and the ability to correct for astigmatism.

2.3. Limited motion mount

As outlined above, two rotation axes are needed for pointing an axisymmetric device, and a third rotation is needed for full reorientation, unless the first two axes are selected with care to provide automatically the correct orientation of the reflector. The LiMoNAED mount described below provides this property of full reorientation with two-axis tracking.

An arbitrary choice of the tracking axes, for example, the common azimuth-elevation drive, leads to varying angular velocities needed to continuously track the sun (Schubnell and Ries, 1990). However, since the motion of the earth is a superposition of two rotations around two well-defined axes with nearly constant angular velocities, it should be possible to track the sun with nearly constant angular velocity by choosing the same natural axes.

Eq. (5), which details the apparent motion of the direction of the sun in a Ptolemaic coordinate system, can be inverted to yield:

$$\vec{h}_0 = O^{-1} \cdot S^{-1} \cdot \vec{h}_t(t). \tag{9}$$

To implement this composite transformation, the first operation is inverting of the earth's spin. The first axis that is fixed with respect to the earth is chosen to be parallel to the polar axis, similar to the traditional polar mount. This allows compensating for the spin of the earth with a constant and opposite angular velocity. The moving side of the first axis has then invariant orientation with respect to the sidereal coordinate system.

The traditional polar mount involves a second declination axis perpendicular to the first. When this polar mount is used with a 'fix-focus' dish, the declination of the sun relative to the system changes seasonally, and the incidence angle on the reflector changes as well. This is since the second axis was chosen arbitrarily and without relation to the seasonal changes due to the orbital motion. In this contribution, we propose a different mount, where the second axis is the normal to the ecliptic plane. Rotation around this axis enables the inversion of the orbital motion as the second operation in Eq. (9).

Fig. 2 shows schematically the relation of the two angular velocities for the daily and annual rotations. The orbital rotation can be decomposed into two components. One component is along the daily rotation: it reduces the effective rate of rotation around the polar axis, and is responsible for the small difference between a sidereal day and a solar day. The second component is perpendicular to the daily rotation, and is responsible for the change of solar declination through the year. In a dish with a conventional polar mount, the reflector and receiver follow the required rotation around the first axis (the receiver is in fact stationary since it is at the axis). However, the

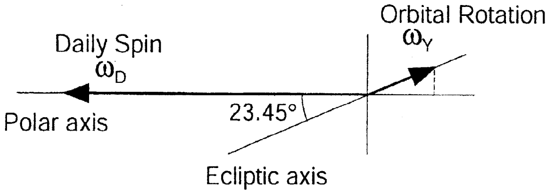

Fig. 2. The angular velocity vectors of the orbital and daily rotations. The orbital angular velocity can be decomposed into components along and perpendicular to the daily spin.

receiver does not follow the motion prescribed by the perpendicular component, and therefore the incidence angle has to change with the seasonal change in declination. In the LiMoNAED mount, both components of the orbital motion are reproduced, and therefore the dish can operate at constant incidence angle regardless of seasonal changes.

3. LIMONAED DESIGN

The new tracking scheme decouples the complex apparent motion of the sun into two stages. The first stage aligns of the line from the dish mount point to the receiver (dish main axis) with the vector normal to the ecliptic plane. A rotation of this line around the polar axis at angular velocity $-\omega_D$ maintains the reflector-to-receiver direction invariant in the sidereal coordinate system. To accomplish this, the dish mount point and the receiver connect to a common rigid platform, which moves such that the dish main axis traces a circular cone (Fig. 3). The cone's half angle is equal to the tilt of the earth's axis, i.e. parallel to the earth's axis. The reflector-to-sun direction is also invariant in the sidereal coordinate system (ignoring for the moment the orbital motion, which is discussed later), always 90° from the direction to the receiver, and the dish incidence angle is a constant 45°.

The reflector rotates together with the tangential plane, and this mount therefore provides a full reorientation of the reflector. Correction of astigmatism by local adjustment of the radii of curvature (Zaibel *et al.*, 1995) is then possible. The flux distribution on the receiver is then circular and axisymmetric. The rotation of the reflector relative to the receiver does not therefore change the incident flux distribution during the day. The only variations are due to scaling with the insolation, similar to an on-axis dish. A given receiver aperture will have then a constant intercept efficiency.

The orbital motion creates a small difference in the length of the sidereal and solar day, leading to a slow drift in the position of the receiver along the circular arc shown in Fig. 3. At summer solstice Noon, the receiver is in the top position, but at winter solstice Noon, it is down at the bottom of the circle. This produces a slow drift in direction of the tangential plane, which depends on the relative direction of the sun $\vec{h}_s(t)$, and is not fixed in the sidereal coordinate system. The second stage provides then a uniform-speed slow rotation of the reflector at angular velocity $-\omega_Y$ about the dish main axis, to compensate for this

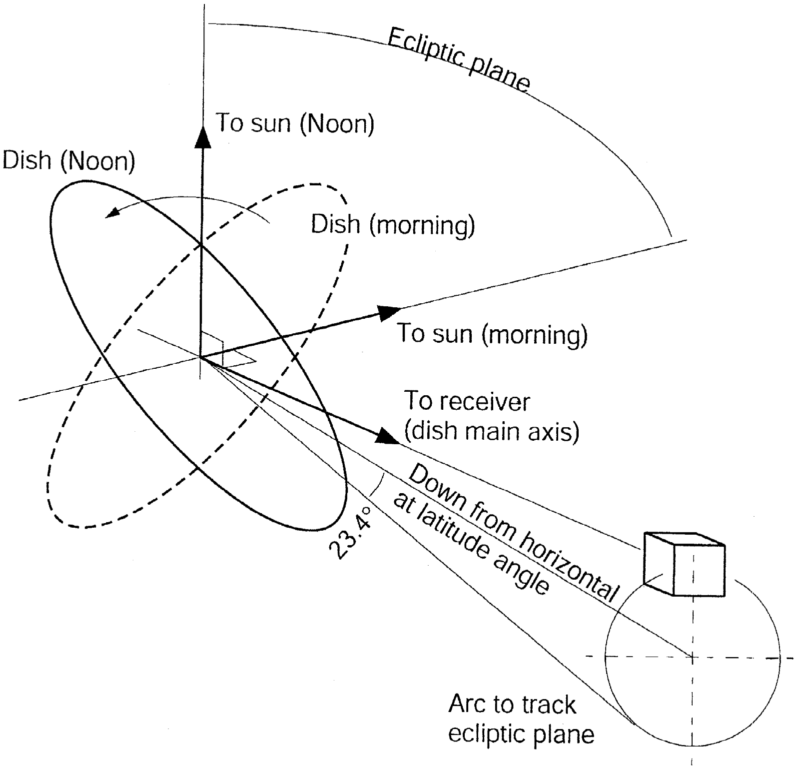

Fig. 3. Ecliptic tracking geometry: the ecliptic tracking arc (first stage), and the rotation of the dish (second stage).

effect. This slow drift redirects the reflected radiation to the correct position of the receiver, and keeps the reflector correctly oriented with respect to the tangential plane.

The off-axis design with 45° incidence angle requires roughly 30% more reflector area per unit collected power, relative to an on-axis dish. Other off-axis designs such as the fix-focus dish also have this property, although the amount of additional reflector area varies with the specific design. The higher system cost due to the additional area should be weighed with other structural and operational features of the different designs, when comparing and selecting the best dish geometry for a particular application.

Implementation of LiMoNAED tracking can be done in many ways, as in any tracking device. One particular implementation is shown in Fig. 4 for the case of a system located in the Northern hemisphere close to the equator, such that the polar axis is nearly horizontal. The dish is supported to the ground with two bearings at the two ends, near the heavy components (reflector and receiver). There is therefore no need for a massive cantilever structure such as used in common designs for dish-Stirling systems. The first stage is a rotation of the entire system around the polar axis. The receiver traces a circular trajectory (two opposite positions on this trajectory are shown in Fig. 4). The radius of the circle is determined by the focal distance. The receiver orientation in space changes by 46.9°, corresponding to twice the polar axis declination from the ecliptic plane. The reflector has an additional rotation around the ecliptic axis, which is performed by a second motor mounted on the first stage. Both motors rotate at a constant speed. The constant speed feature is similar to a polar mount, where the daily rotation is at constant speed (but the day-to-day correction is not).

We have assumed that the rotation is centered on the reflector, since it is usually the most massive element in the system. Therefore, it is preferable to permit the reflector to rotate but not to translate, and the receiver is forced to perform a limited translation. It is possible, however, to reverse the roles: the receiver can rotate without changing the position of the center of its aperture, and the reflector mount point then swings around it, inscribing a circular trajectory corresponding to the cone with half-angle of 23.45°. The properties of alignment with the ecliptic vectors, constant rotation etc., are preserved in this design. This version of the LiMoNAED achieves a stationary receiver that is subject to rotation only, and the rotation is limited within a cone with half-angle of 23.45°.

A similar situation occurs in the 'fix-focus' dish, where the reflector swings around the stationary receiver. However, in this design, the reflector follows a large arc (for example, 180° during equinox). The LiMoNAED mount limits

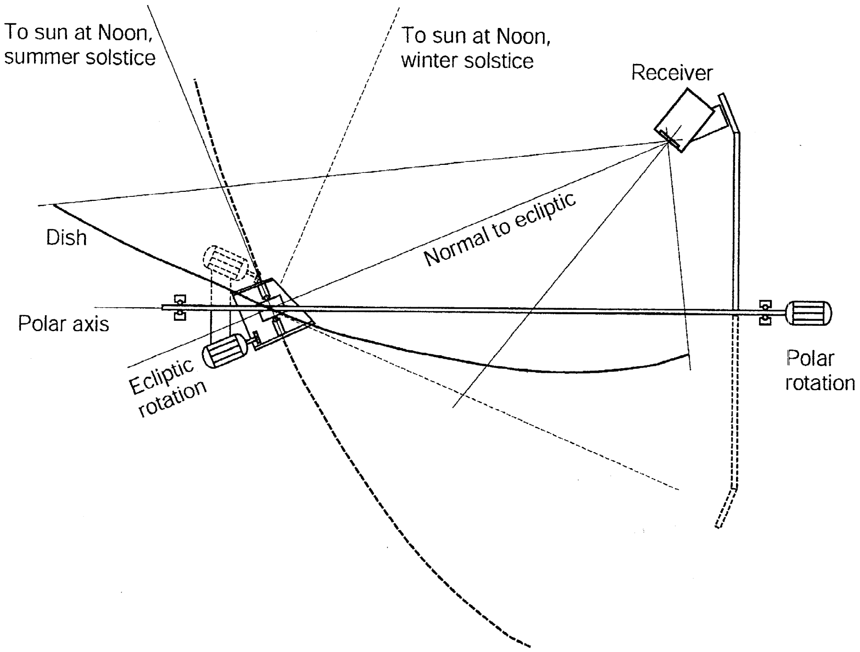

Fig. 4. A possible implementation of LiMoNAED tracking near the equator. Solid line, position at Noon, summer solstice. Dashed line, position at Noon, winter solstice.

the swing of the reflector to twice the declination of the polar axis of the earth.

It is also possible to distribute the translation motion between the receiver and the reflector, by locating the fixed mount point somewhere along the polar axis between them. This option might be considered during detailed design as a compromise between the requirement to limit the receiver motion, and the mechanical balancing of the system.

4. DISCUSSION

A new limited-motion design for a concentrating dish was presented. This design is advantageous when the receiver is not permitted to follow the full arc trajectory that occurs in conventional tracking with an on-axis dish. Relevant applications could be, for example, gas turbines that are sensitive to orientation due to side-forces on bearings or lubricant drain; and chemical reactors that rely on gravity for integrity or effectiveness of the process. Implementation of the new tracking method seems feasible, and an example of a possible design was provided; however, there are many ways to implement any tracking system and better mechanical arrangements may be found. The LiMoNAED implementation presented here uses two rotational drives that move at constant speed. This could simplify the control and improve system reliability in unattended operation. A degree of flexibility in the design allows distributing the limited translation between the reflector and the receiver, according to the specific application. It is therefore possible to design a LiMoNAED with a receiver having a fixed position and only rotational motion. The off-axis design eliminates shading of the dish by the receiver, but requires a larger amount of reflective area for the same power level relative to a conventional on-axis design.

The configuration shown here refers to the case where the receiver is lower than the reflector. This was selected since for some candidate receivers such as solar chemical reactors it is convenient to install the receiver as close as possible to the ground. However, it is also possible to invert the system and install the receiver at the higher end of the polar axis, with the reflector close to the ground. The reflector is then constructed from the opposite side of the off-axis paraboloid. This can be used, for example, in the case of a dish-turbine system where the main motivation was to limit the changes in orientation of the turbine relative to gravity, but electrical connections of the turbine to the ground can be easily extended to an elevated position. In this case, the reflector support may be simpler due to the lower elevation.

The LiMoNAED may be less convenient at high latitude, since the polar axis points down into the ground at the latitude angle. The dish will therefore be elevated by a significant amount relative to the receiver (or the other way around), needing a tall and awkward mechanical support. For example, consider a system installed near the North Pole at summer solstice as an extreme case. This can be imagined by rotating Fig. 4 by 90°. Providing a stable support for the upper bearing will be difficult. In addition, the ecliptic tracking needs to rotate the reflector by 360° since the sun is visible throughout the day. However, this is not possible since the reflector will clash with the upper mount point support. It may be possible to circumvent this problem by using the receiver-on-top version, and providing a cantilever support along the polar axis, rather than a separate support linking the upper end of the axis to the ground. This could be feasible without a massive structure since the cantilever is nearly vertical and will be subject to relatively small side forces. Other systems that use a polar mount with an off-axis reflector will face the same issue.

NOMENCLATURE

$\vec{e}$	unit vector in direction of ecliptic axis
$\vec{h}_s$	unit vector in direction from the earth to the sun in the sidereal coordinate system
$\vec{h}_t$	unit vector in direction from the earth to the sun in the Ptolemaic coordinate system
M	reorientation operator
O, S	orbital motion and spin (daily rotation) motion of the earth
$\vec{p}$	unit vector in direction of earth's polar axis
R	rotation operator
t	time (s)
T_{SD}	duration of solar day (s)
T_Y, T_D	annual and daily rotation periods in sidereal coordinate system (s)
ω_Y, ω_D	annual and daily rotation angular speeds (s^{-1})

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